Optimum Reactive Power Dispatch for Alleviation of Voltage Deviations

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Abstract

This paper presents a non-linear optimization algorithm for alleviation of under-voltage and over-voltage conditions in the day-to-day operation of power networks. Voltage control for varying load and generation conditions can be achieved by coordinated control of switchable shunt VAR compensating (SVC) devices, on load transformer taps (OLTC) and generators excitation. The proposed algorithm for voltage control uses a non-linear least square minimization technique. Results obtained for 6-Bus Ward-Hale system and a modified IEEE 30-Bus system are presented for illustration purposes.

Keywords: least square minimization, OLTC, optimization, SVC, voltage control

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The main objective of this paper is to detail a developed linear programming approach for least squares formulation of optimum reactive power dispatch. Results obtained for 6-Bus Ward-Hale system and a modified IEEE 30-Bus system are presented for illustration purposes.

2. Formulation of Optimization Problem

The optimization technique used is Least square minimization. The objective function used is minimization of sum of the squares of voltage deviations from pre-selected desired values. The control variables considered are switchable shunt reactive power (SVC), OLTC transformers and generators excitation.

Consider a system where,

- \( n \): total number of buses
- \( 1, 2, \ldots, g \): generator buses
- \( g+1, g+2, \ldots, g+s \): SVC buses
- \( g+s+1, \ldots, n \): the remaining buses
- \( t \): number of on load tap changing transformer.

The objective function is expressed as

\[
\min \quad J(X) = \sum_{i=g+1}^{n} \left[ V_{i}^{\text{des}} - V_{i}^{\text{cal}} \right]^2
\]

where \( X \) is the vector of control variables

\[
[X]^t = [\Delta T_1, \ldots, \Delta T_t, \Delta V_1, \ldots, \Delta V_g, \Delta Q_{g+1}, \ldots, \Delta Q_{g+s}]
\]

The condition for minimization of \( J(X) \) is \( \nabla_x J(X) = 0 \). Defining

\[
[H] = \begin{bmatrix}
\frac{\partial V_{g+1}}{\partial T_1} & \ldots & \frac{\partial V_{g+1}}{\partial T_t} & \frac{\partial V_{g+1}}{\partial V_1} & \ldots & \frac{\partial V_{g+1}}{\partial Q_{g+1}} & \frac{\partial V_{g+1}}{\partial Q_{g+s}} \\
\frac{\partial V_{g+2}}{\partial T_1} & \ldots & \frac{\partial V_{g+2}}{\partial T_t} & \frac{\partial V_{g+2}}{\partial V_1} & \ldots & \frac{\partial V_{g+2}}{\partial Q_{g+1}} & \frac{\partial V_{g+2}}{\partial Q_{g+s}} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{\partial V_n}{\partial T_1} & \ldots & \frac{\partial V_n}{\partial T_t} & \frac{\partial V_n}{\partial V_1} & \ldots & \frac{\partial V_n}{\partial Q_{g+1}} & \frac{\partial V_n}{\partial Q_{g+s}}
\end{bmatrix}
\]

We have

\[
\nabla_x J(X) = -2[H]^t [V_{g+1}^{\text{des}} - V_{g+1}^{\text{cal}}] \\
\vdots \\
[V_n^{\text{des}} - V_n^{\text{cal}}]
\]

To make \( \nabla_x J(X) \) equal zero, Newton’s method is applied which gives the corrections required for the control variables

\[
\Delta X = \left[ \frac{\partial^{2} J(X)}{\partial X \partial X} \right]^{-1} \left[ -\nabla_x J(X) \right]
\]

The Jacobian of \( \nabla_x J(X) \) is calculated by treating \( H \) as constant matrix.
\[
\frac{\partial V_g J(X)}{\partial X} = \frac{\partial}{\partial X} \left[ -2[H] \right] \begin{bmatrix}
V_{g+1}^{des} - V_{g+1}^{cal} \\
\vdots \\
V_n^{des} - V_n^{cal}
\end{bmatrix}
\]

\[
\frac{\partial V_g J(X)}{\partial X} = 2[H] [H] 
\]  

(5)

Hence, substituting Equations (3) and (5) in (4), we obtain,

\[
\Delta X = \left[ 2[H] \right]^{-1} \left[ 2[H] \right]^{-1} \begin{bmatrix}
V_{g+1}^{des} - V_{g+1}^{cal} \\
\vdots \\
V_n^{des} - V_n^{cal}
\end{bmatrix}
\]

\[
[H'] [H] \Delta X = [H'] \begin{bmatrix}
V_{g+1}^{des} - V_{g+1}^{cal} \\
\vdots \\
V_n^{des} - V_n^{cal}
\end{bmatrix}
\]  

(6)

2.1. Computation of H Matrix

The element of H matrix cannot be defined directly and so is evaluated as sensitivity matrix. The relation between the net reactive power change at any bus due to change in the transformer tap setting and voltage magnitudes can be written as,

\[
\begin{bmatrix}
\Delta Q_G \\
\Delta Q_S \\
\Delta Q_R
\end{bmatrix} = 
\begin{bmatrix}
A_1 & A_2 & A_3 & A_4 \\
A_5 & A_6 & A_7 & A_8 \\
A_9 & A_{10} & A_{11} & A_{12}
\end{bmatrix} 
\begin{bmatrix}
\Delta T_F \\
\Delta V_G \\
\Delta V_S \\
\Delta V_R
\end{bmatrix}
\]

(7)

where

\[
\Delta Q_G = [\Delta Q_1, ..., \Delta Q_g]^t \\
\Delta Q_S = [\Delta Q_{g+1}, ..., \Delta Q_{g+s}]^t \\
\Delta Q_R = [\Delta Q_{g+s+1}, ..., \Delta Q_n]^t \\
\Delta T_F = [\Delta T_1, ..., \Delta T_l]^t \\
\Delta V_G = [\Delta V_1, ..., \Delta V_g]^t \\
\Delta V_S = [\Delta V_{g+1}, ..., \Delta V_{g+s}]^t \\
\Delta V_R = [\Delta V_{g+s+1}, ..., \Delta V_n]^t
\]

The sub matrices \( A_1 \) to \( A_{12} \) are the corresponding terms of partial derivatives \( \frac{\partial Q}{\partial T} \) and \( \frac{\partial Q}{\partial V} \). Transferring the control variable to the RHS and dependent variables to the LHS we obtain,

\[
\begin{bmatrix}
\Delta Q_G \\
\Delta V_R \\
\Delta Q_R
\end{bmatrix} = 
\begin{bmatrix}
S_1 & S_2 & S_3 \\
S_3 & S_4 & S_5
\end{bmatrix} 
\begin{bmatrix}
\Delta T_F \\
\Delta V_G \\
\Delta Q_S
\end{bmatrix}
\]

(8)

where

\[
S_1 = [B_1] + [-B_2][B_1][B_4]^{-1}[B_3]
\]

(9)
\[
S_2 = [B_2][B_4]^{-1}[B_5] \\
H = [S_3, S_4] \\
S_3 = [B_4]^{-1}[B_1] \\
S_4 = [B_4]^{-1}[B_5] \\
B_1 = [A_i, A_2], \quad B_2 = [-A_3, -A_4] \\
B_3 = [A_5, A_6, A_7, A_{10}] \\
B_4 = [-A_7, -A_8, -A_{11}, -A_{12}] \\
B_5 = \begin{bmatrix}
      -I \\
      0
    \end{bmatrix},
\]

Size of various sub-matrices are:

- \( S_1 \): \((g) \times (t + g)\),
- \( S_2 \): \((g) \times (s)\),
- \( S_3 \): \((s + r) \times (t + g)\),
- \( S_4 \): \((s + r) \times (s)\),
- \( H \): \((s + r) \times (t + g + s)\),
- \( B_1 \): \((g) \times (t + g)\),
- \( B_2 \): \((g) \times (s + r)\),
- \( B_3 \): \((s + r) \times (t + g)\),
- \( B_4 \): \((s + r) \times (s + r)\),
- \( B_5 \): \((s + r) \times (s)\)

\( I \) : is an identity matrix of size \((s \times s)\).

Matrices \( S_3 \) and \( S_4 \) are voltage sensitivities of load and matrices \( S_1 \) and \( S_2 \) are sensitivities of generator Q injections to different reactive power controllers.

### 2.2. Algorithmic Steps

In day-to-day operational power systems, for a particular load and set of network conditions, an optimal combination of real power generation schedule has to be obtained from an active power optimization algorithm. The control variables are to be initialized in the P-optimization algorithm. The following steps are followed to obtain the optimal reactive power allocation in the system.

**Step 1:** Read the system data.
**Step 2:** Form network matrices.
**Step 3:** Perform initial power flow (assumed available from state estimator).
**Step 4:** Compute the voltage error vector
\[
V_{err} = [V_{des} - V_{cal}]
\]
**Step 5:** If all the voltage errors are within the specified tolerance go to step 11.
**Step 6:** Compute \([H]\) matrix using Equation (11).
**Step 7:** Solve for control variables using Equation (6).
**Step 8:** The control variables are adjusted for a suitable step size.
**Step 9:** Control variables are updated and checked for their limits. If no scope for Controller change exist then go to step 11.
**Step 10:** Perform power flow and go to step 1.
**Step 11:** Print the results.

### 2.3. Hard and Soft Constraints

Equipment constraints including SVC, OLTC settings and generator outputs should not exceed its rating due to equipment safety and other operational constraints. Hence SVC, OLTC settings and generators excitation/Q outputs are treated as hard constraints. In case of any voltage violations exist in the system than they must be completely alleviated, if possible, else reduced by suitable control action. Hence the system voltage is considered as a soft constraint.
3. Test System Studies
3.1. Ward-Hale 6-Bus system

The single line diagram of Ward-Hale 6-Bus system, transformers, line data, load data, generation schedule data and SVC setting are adopted from [5]. The controller variable parameters are given in Table-1.

The initial load flow result with nominal VAR control settings are presented in Table-2 from which it is seen that there are 4 voltage bus are not within the desired limits. The system real power losses are 12.91 MW. This situation has been improved by the application of the algorithm proposed in this paper. During the VAR control iteration, the limits on the control variables are taken as follows:

- Transformer tap settings : ± 0.0250 p.u.
- Generator excitation settings : ± 0.0250 p.u, ± 0.01 p.u.
- Switchable VAR compensator settings : ± 1.00 MVAR.

### Table 1. Ward-hale 6-bus system: controller settings

<table>
<thead>
<tr>
<th>Controller Variables</th>
<th>Bus</th>
<th>Initial</th>
<th>VAR control iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From</td>
<td>To</td>
<td>1</td>
</tr>
<tr>
<td>Trf. tap-Setting</td>
<td>6</td>
<td>5</td>
<td>1.0000</td>
</tr>
<tr>
<td>Generator</td>
<td>4</td>
<td>3</td>
<td>1.0000</td>
</tr>
<tr>
<td>Excitation</td>
<td>2</td>
<td></td>
<td>1.0000</td>
</tr>
<tr>
<td>SVC (Q)</td>
<td>4</td>
<td>5</td>
<td>0.0000</td>
</tr>
<tr>
<td>(MVAR)</td>
<td>5</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

### Table 2. Ward-hale 6-bus system: bus voltage magnitude (p.u)

<table>
<thead>
<tr>
<th>Load Bus No.</th>
<th>Initial</th>
<th>VAR control iteration (Optimum)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>1.0250</td>
</tr>
<tr>
<td>3</td>
<td>0.835</td>
<td>0.8350</td>
</tr>
<tr>
<td>4</td>
<td>0.857</td>
<td>0.8750</td>
</tr>
<tr>
<td>5</td>
<td>0.806</td>
<td>0.8150</td>
</tr>
<tr>
<td>6</td>
<td>0.836</td>
<td>0.8550</td>
</tr>
<tr>
<td>P_loss, MW</td>
<td>12.91</td>
<td>11.28</td>
</tr>
</tbody>
</table>

### Table 3. Summary results of ward-hale 6-bus system

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial Value</th>
<th>Optimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MW</td>
<td>MVAR</td>
</tr>
<tr>
<td>Total Generation</td>
<td>147.91</td>
<td>80.58</td>
</tr>
<tr>
<td>Total P - Q Load</td>
<td>135.00</td>
<td>36.60</td>
</tr>
<tr>
<td>Total Power Loss</td>
<td>12.91</td>
<td>44.56</td>
</tr>
<tr>
<td>Total Reactive Compensation (MVAR)</td>
<td>0.00</td>
<td>3.00</td>
</tr>
<tr>
<td>Percentage Power Losses (%)</td>
<td>8.73</td>
<td>7.53</td>
</tr>
<tr>
<td>Reduction in Losses (%)</td>
<td>0.00</td>
<td>13.74</td>
</tr>
<tr>
<td>V_{min} (p.u)</td>
<td>V_S = 0.806</td>
<td>V_S = 0.902</td>
</tr>
<tr>
<td>V_{max} (p.u)</td>
<td>V_{L,S} = 1.000</td>
<td>V_S = 1.125</td>
</tr>
</tbody>
</table>

At the end of 6th iteration, the voltages at all the buses are close within the specified limits and the system losses have also been reduced to 10.99 MW, there by resulting in 13.74% reduction in the real power losses. The results obtained at the end of each VAR control iteration are presented in Tables-1 and 2. The summary results of Hard-Wale 6-bus system are shown in Table-3. The system voltage profile for initial and optimum condition is shown in Figure 1.
3.2. Modified IEEE 30-Bus System

The single line diagram of modified IEEE 30-Bus system, transformers, line data and load data are adopted from [7]. The system has 4 numbers of transformers, 6 numbers of generator buses and 9 numbers of switchable VAR compensation buses. The proposed VAR optimization technique with Voltage alleviation objective is implemented. The controller settings of the parameters are given in Table-4. Voltage profiles of the system after implementing the voltage alleviation techniques are shown in Table-5.

![Graph showing voltage profiles of ward-hale 6-bus system](image_url)

**Figure 1. Voltage profiles of ward-hale 6-bus system**

<table>
<thead>
<tr>
<th>Table 4. Modified IEEE 30-bus system: controller settings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Controller variables</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Transformer tap-Setting</td>
</tr>
<tr>
<td>Generator Excitation</td>
</tr>
<tr>
<td><strong>SVC (Q)</strong> (MVAR)</td>
</tr>
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<tr>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5. Modified IEEE 30-Bus test system: Bus Voltage magnitude (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bus No.</strong></td>
</tr>
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</tbody>
</table>
The initial load flow for this system shows that the voltages at about 17 buses are not within the acceptable operating voltage limits and the system real power losses are about 25.70 MW. The proposed algorithm has been applied to improve the situation. At the end of 6th VAR control iteration, the voltage at all the buses are close within the specified limits and the system real power losses have come down to 20.17 MW, thereby resulting in 21.52% reduction in the real power losses. The system voltage profiles of the system for initial and optimum conditions are shown in Figure 2. The summary results of modified IEEE 30-bus system are shown in Table-6. During the VAR control iterations the limits on the control variables considered are:

- Transformer tap settings: ± 0.025 p.u.
- Generator excitation settings: ± 0.025 p.u.
- Switchable VAR compensation settings: ± 1.0 MVAR.

![Figure 2. Voltage profiles of modified IEEE 30-bus system](image)
Table 6. Summary results of modified IEEE 30-bus system

<table>
<thead>
<tr>
<th></th>
<th>Initial Value</th>
<th>Optimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MW</td>
<td>MVAR</td>
<td>MW</td>
</tr>
<tr>
<td>Total Generation</td>
<td>266.30</td>
<td>238.95</td>
</tr>
<tr>
<td>Total P - Q Load</td>
<td>240.63</td>
<td>130.90</td>
</tr>
<tr>
<td>Total Power Loss</td>
<td>25.70</td>
<td>108.02</td>
</tr>
<tr>
<td>Total Reactive Compensation (MVAR)</td>
<td>0.00</td>
<td>43.00</td>
</tr>
<tr>
<td>Percentage Power Losses (%)</td>
<td>9.64</td>
<td>7.62</td>
</tr>
<tr>
<td>Reduction in Losses (%)</td>
<td>-</td>
<td>15.02</td>
</tr>
<tr>
<td>(V_{\text{min}}) (p.u)</td>
<td>(V_{30} = 0.733)</td>
<td>(V_{30} = 0.815)</td>
</tr>
<tr>
<td>(V_{\text{max}}) (p.u)</td>
<td>(V_{1.4} = 1.000)</td>
<td>(V_{2} = 1.053)</td>
</tr>
</tbody>
</table>

4. Conclusions

A non-linear optimization algorithm employing least squares minimization technique for voltage improvement is proposed. A prototype of an expert system for alleviation of network voltage violations is also developed. The expert system has been tested with simulated conditions of a few practical systems and is demonstrated to give acceptable results in real time when compared to optimization technique proposed in Lomi [5] and curtailed number and reduced controller movement algorithm. From the results obtained we see that the expert system [6], [7] tries to alleviate the voltage violations using minimum number of controllers.

References